Straus, Joseph N. Introduction to Post-Tonal Theory, Upper Saddle River: Pearson Prentice Hall, 2005. Print.

Introduction to Post-Tonal Theory

third edition

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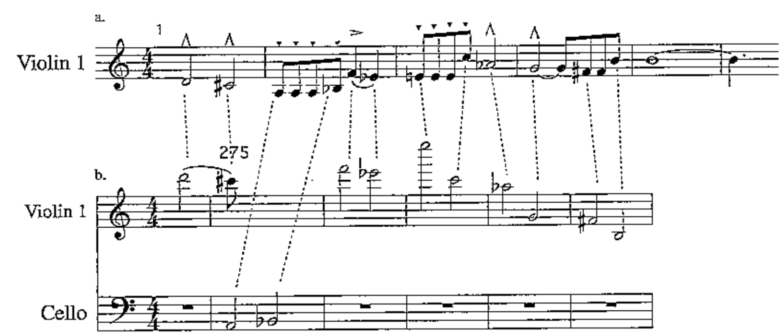
Chapter 1

Basic Concepts and Definitions

Octave Equivalence

There is something special about the octave. Pitches separated by one or more octaves are usually perceived as in some sense *equivalent.* Our musical notation reflects that equivalence by giving the same name to octave-related pitches. The name A, for example, is given not only to some particular pitch, like the A a minor third below middle C, but also to all the other pitches one or more octaves above or below it. Octave-related pitches are called by the same name because they sound so much alike and because Western music usually treats them as functionally equivalent.

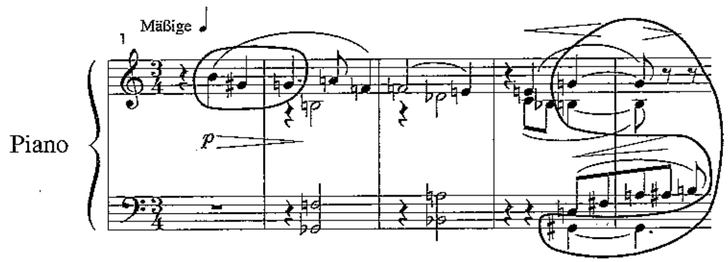
Equivalence is not the same thing as identity. Example 1–1 shows a melody from Schoenberg's String Quartet No. 4, first as it occurs at the beginning of the first movement and then as it occurs a few measures from the end.



Example 1–1 Two equivalent melodies (Schoenberg, String Quartet No. 4).

The two versions are different in many ways, particularly in their rhythm and range. The range of the second version is so wide that the first violin cannot reach all of the notes; the cello has to step in to help. At the same time, however, it is easy to recognize that they are basically the same melody—in other words, that they are octave equivalent.

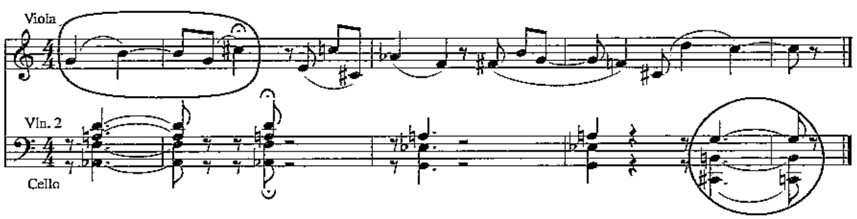
In Example 1–2, the opening of Schoenberg's Piano Piece, Op. 11, No. 1, compare the first three notes of the melody with the sustained notes in measures 4–5.



Example 1–2 Two equivalent musical ideas (Schoenberg, Piano Piece, Op. 11, No. 1).

There are many differences between the two collections of notes (register, articulation, rhythm, etc.), but a basic equivalence also. They are equivalent because they both contain a B, a G♯, and a G.

We find the same situation in the passage shown in Example 1–3, from a string quartet movement by Webern. The first three notes of the viola melody—G, B, and C♯—return as the cadential chord at the end of the phrase. The melody and the chord are octave equivalent.



Example 1–3 Two equivalent musical ideas (Webern, Movements for String Quartet, Op. 5, No. 2).

When we assert octave equivalence, and other equivalences we will discuss later, our object is not to smooth out or dismiss the variety of the musical surface. Rather, we seek to discover the relationships that underlie the surface and lend unity and coherence to musical works.

Pitch Class

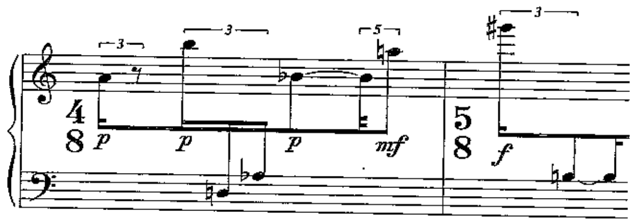
We will distinguish between a *pitch* (a tone with a certain frequency) and a *pitch class* (a group of pitches with the same name). Pitch-class A, for example, contains all the pitches named A. To put it the other way around, any pitch named A is a member of pitch-class A. Sometimes we will speak about specific pitches; at other times we will talk, more abstractly, about pitch classes. When we say that the lowest note on the cello is a C, we are referring to a specific pitch. We can notate that pitch on the second ledger line beneath the bass staff. When we say that the tonic of Beethoven's Fifth Symphony is C, we are referring not to some particular pitch C, but to *pitch-class* C. Pitch-class C is an abstraction and cannot be adequately notated on musical staves. Sometimes, for convenience, we will represent a pitch class using musical notation. In reality, however, a pitch class is not a single thing; it is a class of things, of pitches one or more octaves apart.

The passage shown in Example 1–4 consists of seventeen three-note chords. The pitches change as the instruments jump around, but each chord contains the same three pitch classes: F♯, G, and A♭ (notice that the violin is playing harmonics that produce a pitch two octaves higher than the filled-in notehead).

Example 1–4 Many pitches, but only three pitch classes: F♯, G, and A♭ (Feldman, *Durations III*, No. 3).

Enharmonic Equivalence

In common-practice tonal music, a B♭ is not the same as an A♯. Even on an equal-tempered instrument like the piano, the tonal system gives B♭ and A♯ different functions and different meanings, representing different degrees of the scale. In G major, for example, A♯ is ♯$\hat{2}$ whereas B♭ is ♭$\hat{3}$, and scale-degrees $\hat{2}$ and $\hat{3}$ have very different musical roles both melodically and harmonically. These distinctions are largely abandoned in post-tonal music, however, where notes that are enharmonically equivalent (like B♭ and A♯) are also functionally equivalent. For example, the passage in Example 1–5 involves three repetitions: the A returns an octave higher, the B returns two octaves lower, and the A♭ returns three octaves higher as a G♯. A♭ and G♯ are enharmonically equivalent.



Example 1–5 Enharmonic equivalence (Stockhausen, *Klavierstuck III*).

There may be isolated moments where a composer notates a pitch in what seems like a functional way (sharps for ascending motion and flats for descending, for example). For the most part, however, the notation is functionally arbitrary, determined by simple convenience and legibility. The melodies in Example 1–6 are enharmonically equivalent (although the first one is much easier to read).

Example 1–6 Enharmonic equivalence.

Integer Notation

Octave equivalence and enharmonic equivalence leave us with only twelve different pitch classes. All the B♯s, C♮s, and D𝄫s are members of a single pitch class, as are all the C♯s and D♭s, all the C𝄪s, Ds, and E𝄫s, and so on. We will often use integers from 0 through 11 to refer to the pitch classes. Figure 1–1 shows the twelve different pitch classes and some of the contents of each.

integer name	*pitch-class content*
0	B♯, C, D𝄫
1	C♯, D♭
2	C𝄪, D, E𝄫
3	D♯, E♭
4	D𝄪, E, F♭
5	E♯, F, G𝄫
6	F♯, G♭
7	F𝄪, G, A𝄫
8	G♯, A♭
9	G𝄪, A, B𝄫
10	A♯, B♭
11	A𝄪, B, C♭

Figure 1–1

We will use a "fixed *do*" notation: the pitch class containing the Cs is arbitrarily assigned the integer 0 and the rest follows from there.

Integers are traditional in music (figured-bass numbers, for example) and useful for representing certain musical relationships. We will never do things to the integers that don't have musical significance. We won't divide integers, because, while dividing 7 into 11 makes numerical sense, dividing G into B doesn't make much musical sense. Other arithmetical operations, however, will prove musically useful. We will, for example, subtract numbers, because, as we will see, subtraction gives us a simple way of talking about intervals. Computing the distance between 7 and 11 by subtracting 7 from 11 makes numerical sense, and the idea of computing the distance between G and B makes musical sense. We will use numbers and arithmetic to model interesting aspects of the music we study. The music itself is not "mathematical" any more than our lives are "mathematical" just because we count our ages in integers. In this book, we will identify pitch classes with either traditional letter notation or integers, whichever seems clearest and easiest in a particular context.

Mod 12

Every pitch belongs to one of the twelve pitch classes. Going up an octave (adding twelve semitones) or going down an octave (subtracting twelve semitones) will just produce another member of the same pitch class. For example, if we start on the E♭ above middle C (a member of pitch class 3) and go up twelve semitones, we end up back on pitch class 3. In other words, in the world of pitch classes, $3 + 12 = 15 = 3$. More generally, any number larger than 11 or smaller than 0 is equivalent to some integer from 0 to 11 inclusive. To figure out which one, just add or subtract 12 (or any multiple of 12). Twelve is called the *modulus,* and our theoretical system frequently will rely upon arithmetic *modulo 12,* for which *mod 12* is an abbreviation. In a mod 12 system, $-12 = 0 = 12 = 24$, and so on. Similarly, -13, -1, 23, and 35 are all equivalent to 11 (and to each other) because they are related to 11 (and to each other) by adding or subtracting 12.

It is easiest to understand these (and other) mod 12 relationships by envisioning a circular clockface, like the one in Figure 1–2.

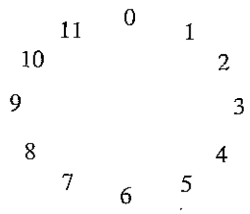


Figure 1–2

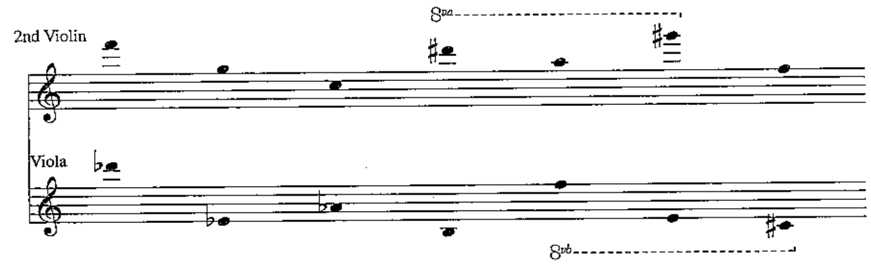
In a mod 12 system, moving 12 (or a multiple of 12) in either direction only brings you back to your starting point. As a result, we will generally be dealing only with integers between 0 and 11 inclusive. When we are confronted with a number larger than 11 or smaller than 0, we will usually write it, by adding or subtracting 12, as an integer between 0 and 11. We will sometimes use negative numbers (for example, when we want to suggest the idea of descending), and we will sometimes use numbers larger than 11 (for example, when discussing the distance between two widely separated pitches), but in general we will discuss such numbers in terms of their mod 12 equivalents.

We locate pitches in an extended *pitch space,* ranging in equal-tempered semitones from the lowest to the highest audible tone. We locate pitch classes in a modular *pitch-class space,* as in Figure 1–2, which circles back on itself and contains only the twelve pitch classes. It's like the hours of the day or the days of the week. As our lives unfold in time, each hour and each day are uniquely located in linear time, never to be repeated. But we can be sure that, if it's eleven o'clock now, it will be eleven o'clock again in twelve hours (that's a mod 12 system), and that if it's Friday today, it will be Friday again in seven days (that's a mod 7 system). Just as our lives unfold simultaneously in linear and modular time, music unfolds simultaneously in pitch and pitch-class space.

Intervals

Because of enharmonic equivalence, we will no longer need different names for intervals with the same absolute size—for example, diminished fourths and major thirds. In tonal music, such distinctions are crucial; intervals are defined and named according to their tonal function. A third, for example, is an interval that spans three steps of the diatonic scale, while a fourth spans four steps. A major third is consonant, while a diminished fourth is dissonant. In music that doesn't use diatonic scales and doesn't systematically distinguish between consonance and dissonance, it seems cumbersome and even misleading to use traditional interval names. It will be easier and more accurate musically just to name intervals according to the number of semitones they contain. The intervals between C and E and between C and F♭ both contain four semitones and are both instances of interval 4, as are B♯–F♭, C–D𝄪, and so on.

Example 1–7 extracts a series of seven harmonic intervals played in rhythmic unison by the second violin and viola in a passage from Elliott Carter's String Quartet No. 3, a piece in which two instrumental duos often play distinct intervals. The first six intervals are spelled as major thirds while the seventh is spelled as a diminished fourth, but in this musical context it is clear that all seven intervals are to be understood as enharmonically equivalent.



Example 1–7 Enharmonically equivalent intervals (Carter, String Quartet No. 3, mm. 245–62).

Figure 1–3 gives some traditional interval names and the number of semitones they contain.

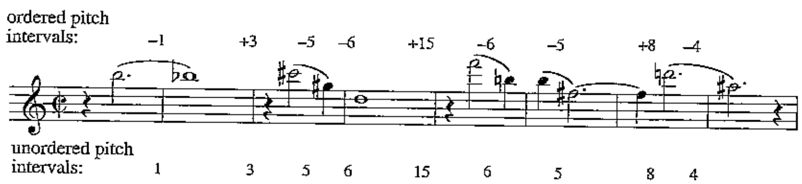
traditional name	*no. of semitones*	*traditional name*	*no. of semitones*
unison	0	major 6th, diminished 7th	9
minor 2nd	1	augmented 6th, minor 7th	10
major 2nd, diminished 3rd	2	major 7th	11
minor 3rd, augmented 2nd	3	octave	12
major 3rd, diminished 4th	4	minor 9th	13
augmented 3rd, perfect 4th	5	major 9th	14
augmented 4th, diminished 5th	6	minor 10th	15
perfect 5th, diminished 6th	7	major 10th	16
augmented 5th, minor 6th	8		

Figure 1–3

Pitch Intervals

A pitch interval is simply the distance between two pitches, measured by the number of semitones between them. A pitch interval, which will be abbreviated *ip,* is created when we move from pitch to pitch in pitch space. It can be as large as the range of our hearing or as small as a semitone. Sometimes we will be concerned about the direction of the interval, whether ascending or descending. In that case, the number will be preceded by either a plus sign (to indicate an ascending interval) or a minus sign (to indicate a descending interval). Intervals with a plus or minus sign are called *directed* or *ordered intervals.* At other times, we will be concerned only with the absolute space between two pitches. For such *unordered intervals,* we will just provide the number of semitones between the pitches.

Whether we consider the interval ordered or unordered depends on our particular analytical interests at the time. Example 1–8 shows the opening melody from Schoenberg's String Quartet No. 3, and identifies both its ordered and unordered pitch intervals.



Example 1–8 Ordered and unordered pitch intervals (Schoenberg, String Quartet No. 3).

The ordered pitch intervals focus attention on the contour of the line, its balance of rising and falling motion. The unordered pitch intervals ignore contour and concentrate entirely on the spaces between the pitches.

Ordered Pitch-Class Intervals

A pitch-class interval is the distance between two pitch classes. A pitch-class interval, which will be abbreviated *i,* is created when we move from pitch class to pitch class in modular pitch-class space. It can never be larger than eleven semitones. As with pitch intervals, we will sometimes be concerned with ordered intervals and sometimes with unordered intervals. To calculate pitch-class intervals, it is best to think again of a circular clockface as in Figure 1–2. We will consider clockwise movement to be equivalent to movement upward, and counterclockwise movement equivalent to movement downward. With this in mind, the ordered interval from C♯ to A, for example, is –4 or +8. In other words, from pitch-class C♯, one can go either up eight semitones or down four semitones to get to pitch-class A. This is because +8 and –4 are equivalent (mod 12). It would be equally accurate to call that interval 8 or –4. By convention, however, we will usually denote ordered pitch-class intervals by an integer from 0 to 11. To state this as a formula, we can say that the ordered interval from pitch class x to pitch class y is y – x (mod 12). Notice that the ordered pitch class interval from A to C♯ (1 – 9 = –8 (mod 12) = 4) is different from that from C♯ to A (8), since, when discussing ordered pitch-class intervals, order matters. Four and 8 are each other's *complement mod 12,* because they add up to 12, as do 0 and 12, 1 and 11, 2 and 10, 3 and 9, and 5 and 7. Six is its own complement mod 12.

Figure 1–4 calculates some ordered pitch-class intervals using the formula.

The ordered pitch-class interval from C♯ to E♭ is 3 – 1 = 2
from E♭ to C♯ is 1 – 3 = 10
from B to F is 5 – 11 = 6
from D to B♭ is 10 – 2 = 8
from B♭ to C♯ is 1 – 10 = 3

Figure 1–4

You will probably find it faster just to envision a musical staff, keyboard, or a clockface. To find the ordered pitch-class interval between C♯ and A, just envision the C♯ and then count the number of half-steps you will need to go upward (if you are envisioning a staff or keyboard) or clockwise (if you are envisioning a clockface) to the nearest A.

Unordered Pitch-Class Intervals

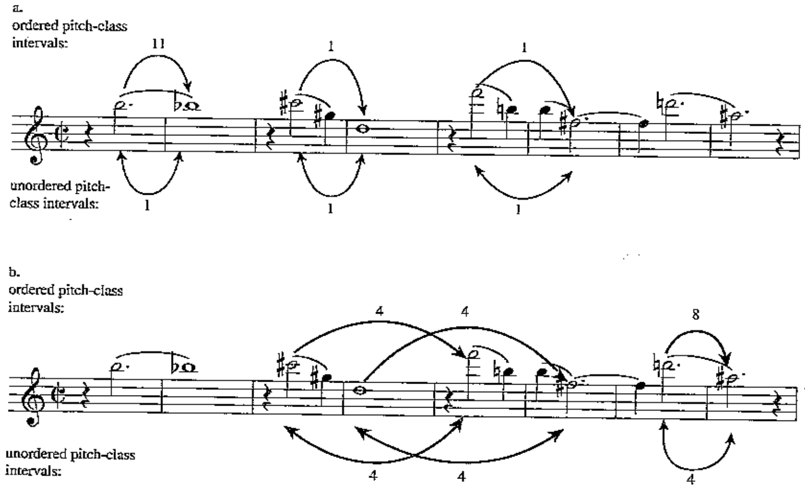
For unordered pitch-class intervals, it no longer matters whether you count upward or downward. All we care about is the space between two pitch classes. Just count from one pitch class to the other by the shortest available route, either up or down. The formula for an unordered pitch-class interval is x – y (mod 12) *or* y – x (mod 12), whichever is smaller. The unordered pitch-class interval between C♯ and A is 4, because 4 (1 – 9 = –8 = 4) is smaller than 8 (9 – 1 = 8). Notice that the unordered pitch-class interval between C♯ and A is the same as that between A and C♯. It is 4 in both cases, since from A to the nearest C♯ is 4 and from C♯ to the nearest A also is 4. Including the unison, 0, there are only seven different unordered pitch-class intervals, because, to get from one pitch class to any other, one never has to travel farther than six semitones. Figure 1–5 calculates some unordered pitch-class intervals using the formula. The correct answer is underlined.

The unordered pitch-class interval between C♯ and E♭ is 3 – 1 = $\underline{2}$ or 1 – 3 = 10
E♭ and C♯ is 1 – 3 = 10 or 3 – 1 = $\underline{2}$
B and F is 5 – 11 = $\underline{6}$ or 11 – 5 = $\underline{6}$
D and B♭ is 10 – 2 = 8 or 2 – 10 = $\underline{4}$
B♭ and C♯ is 1 – 10 = $\underline{3}$ or 10 – 1 = 9

Figure 1–5

Again, you will probably find it faster just to envision a clockface, musical staff, or keyboard. To find the unordered pitch-class interval between B♭ and F♯, for example, just envision a B♭ and count the number of semitones to the nearest available F♯ (4).

In Example 1–9a (again the opening melody from Schoenberg's String Quartet No. 3), the first interval is ordered pitch-class interval 11, to be abbreviated as i11.



Example 1–9 Ordered and unordered pitch-class intervals (Schoenberg, String Quartet No. 3).

That's because to move from B to B♭ one moves –1 or its mod 12 equivalent, i11. Eleven is the name for descending semitones or ascending major sevenths or their compounds. If the B♭ had come before the B, the interval would have been i1, which is the name for ascending semitones or descending major sevenths or their compounds. And that is the interval described by the two subsequent melodic gestures, C♯–D and F–F♯. As ordered pitch-class intervals, the first is different from the second and third. As unordered pitch-class intervals, all three are equivalent. In Example 1–9b, two statements of i4 are balanced by a concluding i8; all three represent unordered pitch-class interval 4.

Interval Class

An unordered pitch-class interval is also called an *interval class*. Just as each pitch-class contains many individual pitches, so each interval class contains many individual pitch intervals. Because of octave equivalence, compound intervals—intervals larger than an octave—are considered equivalent to their counterparts within the octave. Furthermore, pitch-class intervals larger than six are considered equivalent to their complements mod 12 (0 = 12, 1 = 11, 2 = 10, 3 = 9, 4 = 8, 5 = 7, 6 = 6). Thus, for

example, intervals 23, 13, 11, and 1 are all members of interval class 1. Figure 1–6 shows the seven different interval classes and some of the contents of each.

interval class	0	1	2	3	4	5	6
pitch intervals	0,12,24	1,11,13	2,10,14	3,9,15	4,8,16	5,7,17	6,18

Figure 1–6

We thus have four different ways of talking about intervals: ordered pitch interval, unordered pitch interval, ordered pitch-class interval, and unordered pitch-class interval. If in some piece we come across the musical figure shown in Example 1–10, we can describe it in four different ways.

ordered pitch interval: +19
unordered pitch interval: 19
ordered pitch-class interval: 7
unordered pitch-class interval: 5

Example 1–10 Four ways of describing an interval.

If we call it a +19, we have described it very specifically, conveying both the size of the interval and its direction. If we call it a 19, we express only its size. If we call it a 7, we have reduced a compound interval to its within-octave equivalent. If we call it a 5, we have expressed the interval in its simplest, most abstract way. None of these labels is better or more right than the others—it's just that some are more concrete and specific while others are more general and abstract. Which one we use will depend on what musical relationship we are trying to describe.

It's like describing any object in the world—what you see depends upon where you stand. If you stand a few inches away from a painting, for example, you may be aware of the subtlest details, right down to the individual brushstrokes. If you stand back a bit, you will be better able to see the larger shapes and the overall design. There is no single "right" place to stand. To appreciate the painting fully, you have to be willing to move from place to place. One of the specially nice things about music is that you can hear a single object like an interval in many different ways at once. Our different ways of talking about intervals will give us the flexibility to describe many different kinds of musical relationships.

Interval-Class Content

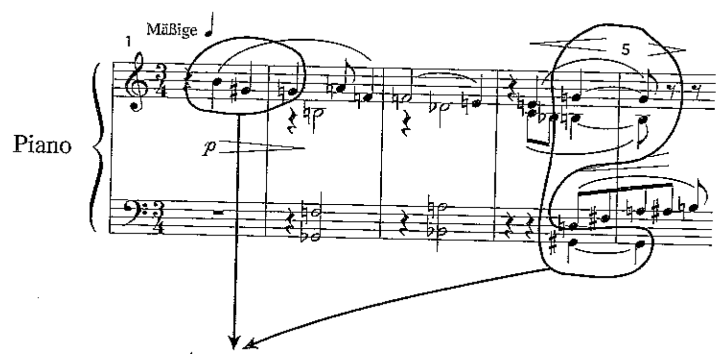
The quality of a sonority can be roughly summarized by listing all the intervals it contains. To keep things simple, we will generally take into account only interval classes (unordered pitch-class intervals). The number of interval classes a sonority contains depends on the number of distinct pitch classes in the sonority. The more pitch classes, the greater the number of interval classes. Figure 1–7 summarizes the number of interval classes in sonorities of all sizes. (We won't bother including the occurrences of interval class 0, which will always be equal to the number of pitch classes in the sonority.)

no. of pitch classes	*no. of interval classes*
1	0
2	1
3	3
4	6
5	10
6	15
7	21
8	28
9	36
10	45
11	55
12	66

Figure 1–7

For any given sonority, we can summarize the interval content in scoreboard fashion by indicating, in the appropriate column, the number of occurrences of each of the six interval classes, again leaving out the occurrences of interval class 0. Such a scoreboard conveys the essential sound of a sonority. Notice that now we are counting all of the intervals in the sonority, not just those formed by notes that are right next to each other. That is because all of the intervals contribute to the overall sound.

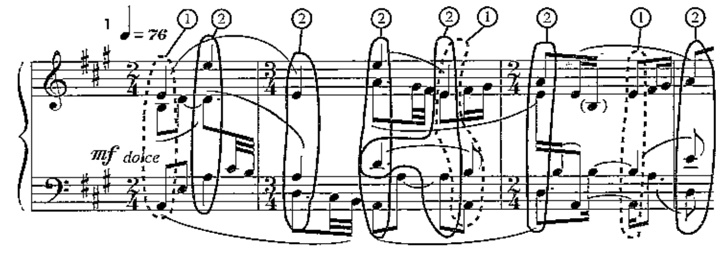
Example 1–11 refers to the same passage and the same three-note sonority discussed back in Example 1–2.

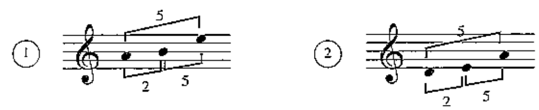


interval class	1	2	3	4	5	6
no. of occurrences	1	0	1	1	0	0

Example 1–11 Interval-class content of a three-note motive (Schoenberg, Piano Piece, Op. 11, No. 1).

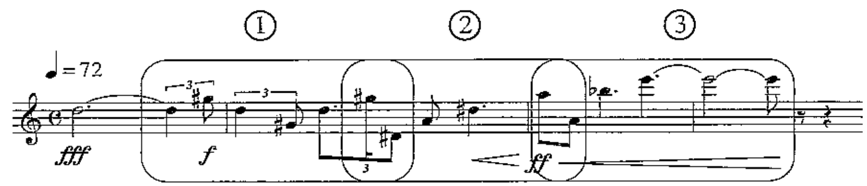
Like any three-note sonority, it contains three intervals, in this case one occurrence each of interval classes 1, 3, and 4 (no 2s, 5s, or 6s). How different this is from the sonorities preferred by Stravinsky in the passage from his opera *The Rake's Progress,* shown in Example 1–12 or by Varèse in the passage from his solo flute piece *Density 21.5* shown in Example 1–13! Stravinsky's chords contain only 2s and 5s and Varèse's melodic cells contain only 1s, 5s, and 6s. The difference in their sound is a reflection of the difference in their interval content.





interval class	1	2	3	4	5	6
no. of occurrences	0	1	0	0	2	0

Example 1–12 Interval-class content of a three-note motive (Stravinsky, *The Rake's Progress,* Act I).



interval class	1	2	3	4	5	6
no. of occurrences	1	0	0	0	1	1

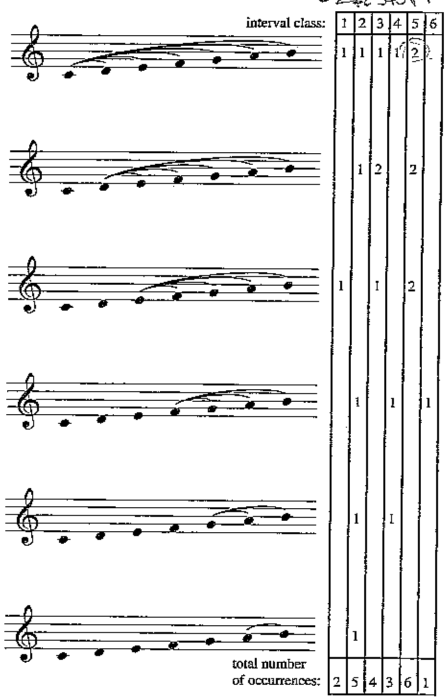
Example 1–13 Interval-class content of a three-note motive (Varese, *Density 21.5,* mm. 11–14).

Interval-Class Vector

Interval-class content is usually presented as a string of six numbers with no spaces intervening. This is called an *interval-class vector.* The first number in an interval-class vector gives the number of occurrences of interval class 1; the second gives the number of occurrences of interval class 2; and so on. The interval-class vector for the sonority in Example 1–11 is 101100, the interval-class vector for the sonority in Example 1–12 is 010020, and the interval-class vector for the sonority in Example 1–13 is 100011.

We can construct a vector like this for sonorities of any size or shape. A tool like the interval-class vector would not be nearly so necessary for talking about traditional tonal music. There, only a few basic sonorities—four kinds of triads and five kinds of seventh chords—are regularly in use. In post-tonal music, however, we will confront a huge variety of harmonies. The interval-class vector will give us a convenient way of summarizing their basic sound.

Even though the interval-class vector is not as necessary a tool for tonal music as for post-tonal music, it can offer an interesting perspective on traditional formations. Example 1–14 calculates the interval-class vector for the major scale.



Example 1–14 Interval-class vector for the major scale.

Notice our methodical process of extracting each interval class. First, the intervals formed with the first note are extracted, then those formed with the second note, and so on. This ensures that we find all the intervals and don't overlook any. As with any seven-note collection, there are 21 intervals in all.

Certain intervallic properties of the major scale are immediately apparent from the interval-class vector. It has only one tritone (fewer than any other interval) and six occurrences of interval-class 5, which contains the perfect fourth and fifth (more than any other interval). This probably only confirms what we already knew about this scale, but the interval-class vector makes the same kind of information available about less familiar collections. The interval-class vector of the major scale has another interesting property—it contains a different number of occurrences of each of the interval classes. This is an extremely important and rare property (only three other collections have it) and it is one to which we will return. For now, the important thing is the idea of describing a sonority in terms of its interval-class content.

BIBLIOGRAPHY

The material presented in Chapter 1 (and in much of Chapters 2 and 3 as well) is also discussed in three widely used books: Allen Forte, *The Structure of Atonal Music* (New Haven: Yale University Press, 1973); John Rahn, *Basic Atonal Theory* (New York: Longman, 1980); and George Perle, *Serial Composition and Atonality,* 6th ed., rev. (Berkeley and Los Angeles: University of California Press, 1991). Two important books offer profound new perspectives on this basic material, and much else besides: David Lewin, *Generalized Musical Intervals and Transformations* (New Haven: Yale University Press, 1987); and Robert Morris, *Composition with Pitch Classes* (New Haven: Yale University Press, 1987). Ambitious students will be interested in Robert Morris, *Class Notes for Atonal Music Theory* (Hanover, N.H.: Frog Peak Music, 1991) and *Class Notes for Advanced Atonal Music Theory* (Hanover, N.H.: Frog Peak Music, 2001). For an aural skills approach to post-tonal theory, see Michael Friedmann, *Ear Training for Twentieth-Century Music* (New Haven: Yale University Press, 1990).

Exercises

THEORY

I. Integer Notation: Any pitch class can be represented by an integer. In the commonly used "fixed *do*" notation, C = 0, C♯ = 1, D = 2, and so on.

1. Represent the following melodies as strings of integers: